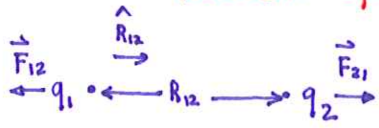
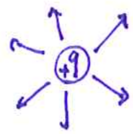
REVIEW of chapter 1, 2, 3

Introduction:

Coulomb's law: 

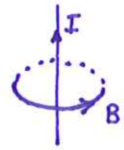
$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r R_{12}^2}\hat{R}_{12} \quad (N)$$

Electric Field: 

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{R} \quad \left(\frac{V}{m}\right)$$

Electric flux Density: $\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$

$\varepsilon_0 = 8.854\times10^{-12}\ \frac{F}{m}$

Biot_Savart law: $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ (T) 

$\mu_0 = 4\pi\times10^{-7}\ \frac{H}{m}$

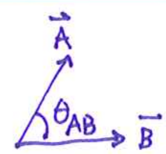
Magnetic field Density: $H = \frac{B}{\mu}$

Sinusoidal wave:

Lossless: $y(x,t) = A\cos(\omega t - \beta x + \phi_0)$ $\quad \omega = \frac{2\pi}{T} = 2\pi f$ & $\beta = \frac{2\pi}{\lambda}$

Lossy: $y(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x + \phi_0)$

Vector Algebra:



$$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\times\vec{B} = AB\sin\theta_{AB}\,\hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\theta_{AB} = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{AB}$$

"bac-cab" rule: $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

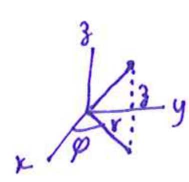
Complex number: $z = x + jy = |z|e^{j\theta}$ $\quad |z| = \sqrt{x^2+y^2}$ $\quad \theta = \tan^{-1}\frac{y}{x}$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$z^n = |z|^n e^{jn\theta}$ $\rightarrow$ Also $\sqrt[n]{z} = \sqrt[n]{|z|}\,e^{j\frac{\theta}{n}}$ (becaus $\sqrt[n]{z} = z^{1/n}$)

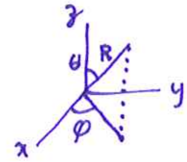
Phasor: $z(x,t) = A\cos(\omega t - \beta x + \phi_0) \leftrightarrow \tilde{z} = Ae^{-j\beta x}e^{j\phi_0}$

Coordinate systems:

Cartesian to cylindrical: $r = \sqrt{x^2+y^2}$ $\quad \varphi = \tan^{-1}\frac{y}{x}$ $\quad z = z$ 

Cylindrical to cartesian: $x = r\cos\varphi \quad y = r\sin\varphi \quad z = z$

Cartesian to spherical: $R = \sqrt{x^2+y^2+z^2} \quad \theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \quad \phi = \tan^{-1}\frac{y}{x}$



Spherical to cartesian: $x = R\sin\theta\cos\varphi \quad y = R\sin\theta\sin\varphi \quad z = R\cos\theta$

$\vec{\nabla}$: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$

Gradient: $\vec{\nabla}A = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)A = \frac{\partial A}{\partial x}\hat{a}_x + \frac{\partial A}{\partial y}\hat{a}_y + \frac{\partial A}{\partial z}\hat{a}_z$ (A is scalar)

Divergence: $\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\cdot\left(A_x a_x + A_y\hat{a}_y + A_z\hat{a}_z\right) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ ($\vec{A}$ is a vector)

Curl: $\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$ ($\vec{A}$ is a vector)

Identities: Divergence of any curl is zero $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

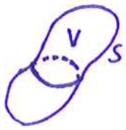
Curl of any gradient is zero $\vec{\nabla}\times(\vec{\nabla}A) = 0$

Laplacian, ∇^2: $\nabla^2 \triangleq \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \rightarrow \nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$ (A can be scalar or vector)

Stokes's Theorem:

$$\int_S (\vec{\nabla}\times\vec{A})\cdot\vec{ds} = \oint_C \vec{A}\cdot\vec{dl}$$

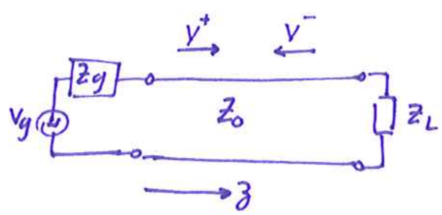
Divergence Theorem:



$$\int_V \vec{\nabla}\cdot\vec{A}\,dv = \int_S \vec{A}\cdot\vec{ds}$$

Transmission Lines:

voltage and current at point z:

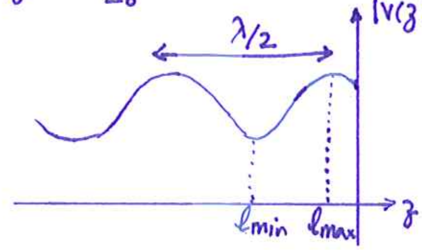


$$\begin{cases} V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right) \end{cases}$$

$\gamma = \underbrace{\alpha}_{\text{Attenuation}} + j\underbrace{\beta}_{\text{Phase constant}}$ if lossless: $\gamma = j\beta$

Reflection Coefficient: $\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$ where $z_L = \frac{Z_L}{Z_0}$ $\quad \Gamma = \frac{V_0^-}{V_0^+} = \frac{-I_0^-}{I_0^+} = |\Gamma| e^{j\theta_r}$

Standing wave: The repetition period is $\frac{\lambda}{2}$ for the standing wave.

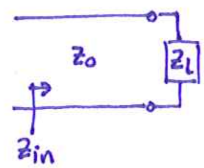


$2\beta z + \theta_r = -2n\pi \Rightarrow l_{max} = -\frac{\theta_r \lambda}{4\pi} - \frac{n\lambda}{2} \quad n = 0, 1, 2 \cdots$

$l_{min} = l_{max} \pm \frac{\lambda}{4}$

Voltage standing wave Ratio: (VSWR or SWR) $S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \geq 1$

Input Impedance: $Z_{in}(l) = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0$



If short circuited line, $Z_L = 0 \rightarrow Z_{in}^{sc}(l) = jZ_0 \tan\beta l$

If open circuited line, $Z_L = \infty \rightarrow Z_{in}^{oc}(l) = -jZ_0 \cot\beta l$

$\Rightarrow Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$

$\tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}}$

Line of $l = n\frac{\lambda}{2}$: $Z_{in} = Z_L$ as everything repeates by $\frac{\lambda}{2}$

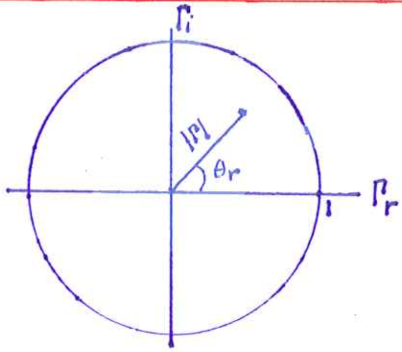
Line of $l = \frac{\lambda}{4} + n\frac{\lambda}{2}$ (quarter-wave transformer): $Z_{in} = \frac{Z_0^2}{Z_L}$

Matched line: $Z_L = Z_0 \rightarrow \Gamma = 0$

Power flow: $P_{av} = P_{av}^i + P_{av}^r = \underbrace{\frac{|V_0^+|^2}{2Z_0}}_{P_{av}^i} - \underbrace{|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}}_{P_{av}^r}$

The Smith Chart:

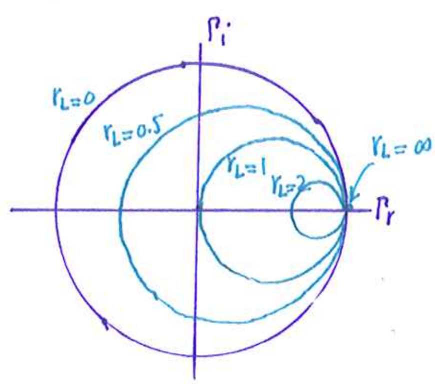
$\Gamma = |\Gamma| e^{j\theta_r}$
$= \Gamma_r + j\Gamma_i$

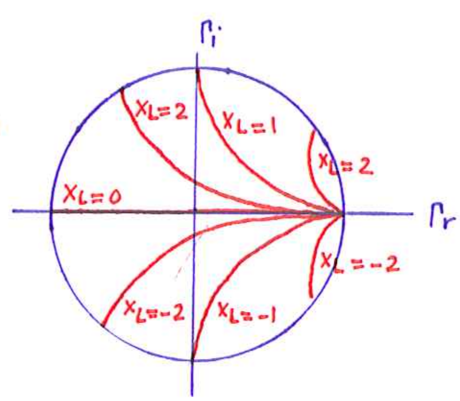


$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} \rightarrow z_L = \frac{1 + \Gamma}{1 - \Gamma} = r_L + jx_L$

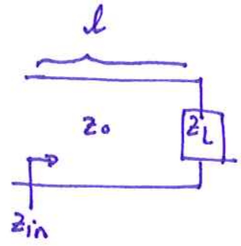
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$

$\Rightarrow z_L = \frac{1 + \Gamma}{1 - \Gamma} = r_L + jx_L$



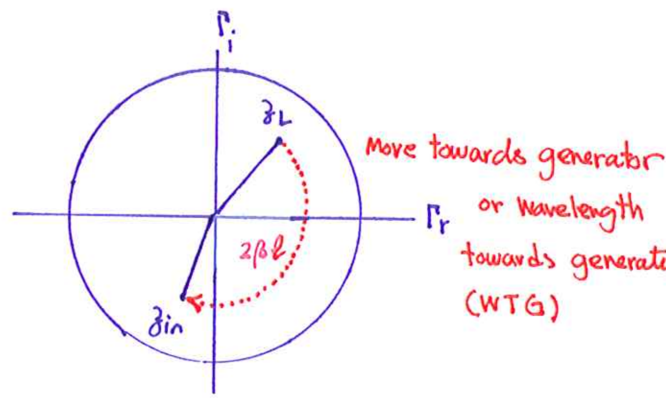


Input impedance:



$$z_L = \frac{Z_L}{Z_o}$$

$$z_{in} = \frac{Z_{in}}{Z_o}$$

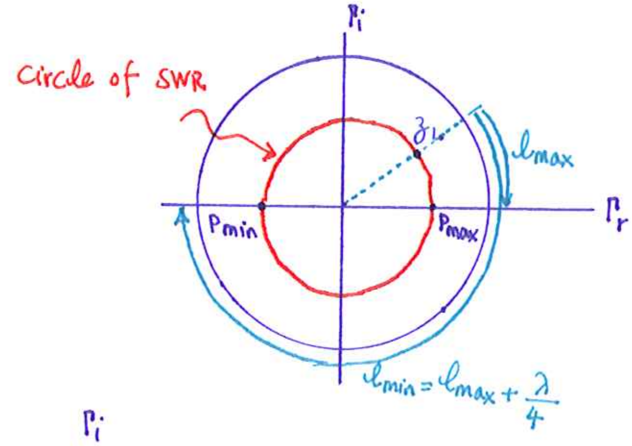


SWR, Maximum and minimum voltage locations:

At max or min points $\Gamma_i = 0 \rightarrow |\Gamma| = \Gamma_r$

Also $z_L = r_L + j\,x_L = r_L \quad (x_L = 0)$

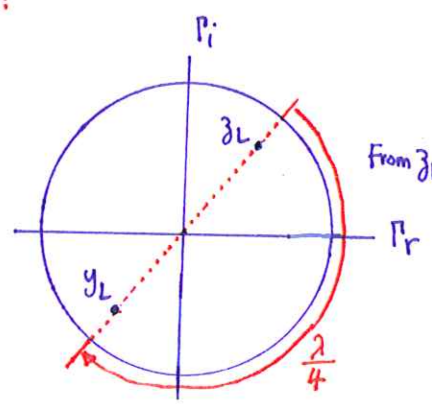
Also at max : $S = r_L$



Impedance to Admittance Transformation:

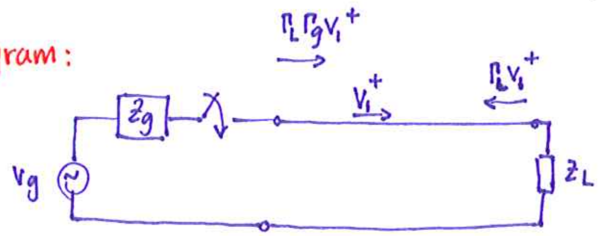
$$y_L = \frac{1}{z_L} = \frac{1-\Gamma}{1+\Gamma}$$

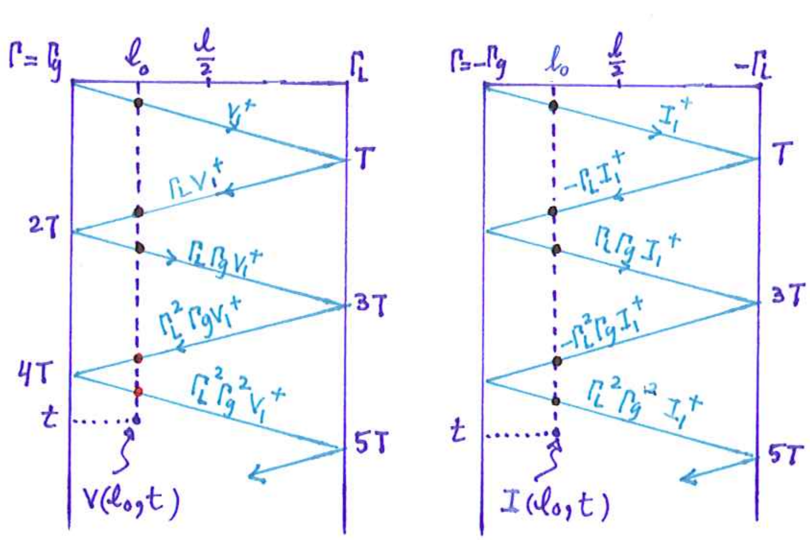
note: $y_L = \frac{Y_L}{Y_o} = Z_o Y_L$



From z_L rotate by $\frac{\lambda}{4}$ to get y_L. This is the opposite point in the chart.

Bounce diagram:





See the notes P. 54 for the case of $l_o = \frac{l}{4}$